SCIENCE FOR GLASS PRODUCTION

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CHOICE OF A MODEL OF SHEET GLASS DEFORMATION IN PRODUCTION OF BENT ARTICLES

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Based on the model of the Institute of Silicate Chemistry, a mathematical model of glass beam deformation is proposed. This model is tested using the experimental data, and its application makes it possible to predict the behavior of glass beam samples in molding.

Multiple attempts to give an adequate mathematical description of the sheet glass deformation process which takes place in production of bent articles often produced rather contradictory results [1 – 3]. In every case there was a question with respect to the accuracy of the analytical calculations which could be only verified experimentally. Such a possibility emerged when researchers at the Belgorod Technological Academy of Construction Materials developed a viscometer which makes it possible to simulate cross-section flexural strain of a beam made from a particular glass composition over a wide temperature range.

The design of the instrument provided for strict observance of the prescribed temperature conditions (with an accuracy of about 2%) and measurement of the current flexural strain of the beam using a mechanotron. Some results of the investigation of window pane plass were kindly presented to us by Yu. L. Belousov (Cand. Sci., Belgorod Technological Academy) and are reported in the present paper.

A preliminary analysis of a number of mathematical models describing processes of temporary deformation of samples at high temperatures indicated that the model of the ISC (Instutute of Silicate Chemistry, Russian Academy of Sciences) developed precisely for glass [4] is the most adequate. However, its authors did not pose the problem of describing the deformation of complex design schemes and limited themselves to the case of axial tension of the beam. In our study, we wanted to test the adequacy of the ISC model for the case of beam flexure under the effect of an integrated load.

In particular, the beam in Fig. 1 is loaded with distributed gravity load q and concentrated load F applied to the center of the span. This design scheme fully corresponds to the deformation of the actual sample in the conditions of the experiment described above.

The maximum value of the sag for purely elastic deformation will be [5]

$$\omega_{\max} = \frac{5ql^4}{384EJ} + \frac{Fl^3}{48EJ},$$

where l is the beam length; E is the elongation modulus; J is the axial moment of inertia of the beam section.

Since the ISC model describes the elongation variation with time τ, i.e.

$$\varepsilon(\tau) = \frac{\sigma(0)}{\eta}\tau + \varepsilon_d(\infty)[1 - M_\varepsilon(\tau)],$$

where $\sigma(0)$ is the initial strain at the time moment $\tau = 0$; η is the material viscosity; $\varepsilon_d(\infty)$ is the creep component of deformation at $\tau \to \infty$; $M_\varepsilon(\tau)$ is the relaxation function

$$M_\varepsilon(\tau) = \left[1 - \exp(-(\tau/\tau_p)^b)\right],$$

where τ is the current time; τ_r is the duration of stress relaxation; $b = 0.5$, the relationship between the maximum sag $\omega_{\max}$ and the static elongation of the lower (stretched) fiber $\varepsilon_{\max}$ was found:

$$\omega_{\max} = \frac{l^2\left(F + \frac{5}{8}ql\right)}{6h\left(F + \frac{1}{2}ql\right)}\varepsilon_{\max},$$

where h is the beam thickness.

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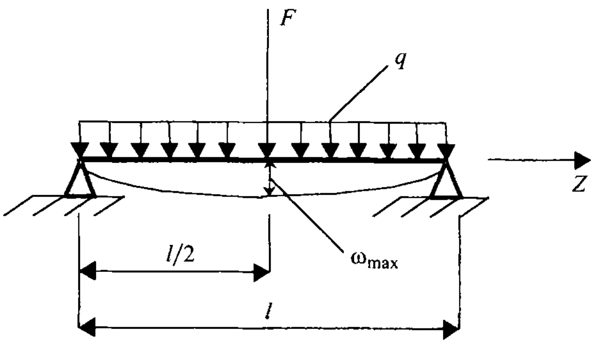

Fig. 1. Scheme of beam deformation in calculation and in experiments.

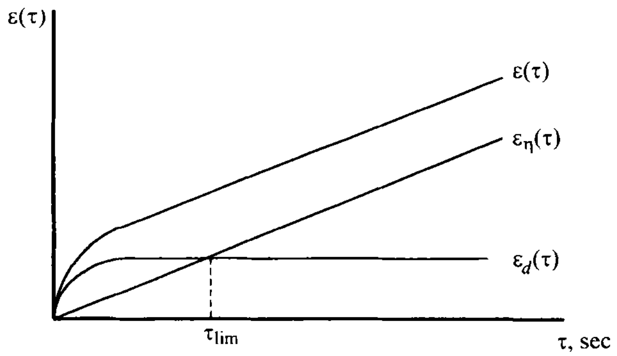

Fig. 2. Dependence of the total deformation on its components.

Thereafter it is sufficiently easy to find the expression for determining the temporal variation in beam sag:

$$\omega_{\max} = \frac{l^2\left(F + \frac{5}{8}ql\right)}{6h\left(F + \frac{1}{2}ql\right)}\left(\frac{\sigma(0)}{\eta}\tau + \varepsilon_d(\infty)[1 - M_\varepsilon(\tau)]\right).$$

Thus, in order to calculate $\omega(\tau)$, it is necessary first to determine $\eta = \eta(t)$, $\varepsilon_d(\infty)$ and $\tau_r = \tau_r(t)$, where t is the temperature of glass. The viscous and relaxation parameters of glass for a given temperature level can be calculated according to O. B. Mazurin [6], and $\varepsilon_d(\infty)$ can be found in the following way.

Let us decompose plot $\varepsilon(\tau)$ into its components in the same coordinate system (Fig. 2). It can be seen that curve $\varepsilon(\tau)$ can be found from the geometric summation of $\varepsilon_d(\tau)$ and $\varepsilon_\eta(\tau)$, where $\varepsilon_\eta(\tau)$ is the viscous deformation component calculated from the formula:

$$\varepsilon_\eta(\tau) = \frac{\sigma(0)}{\eta}\tau.$$

With an increase in τ at fixed temperature and strain values, function $\varepsilon_d(\tau)$ approaches a certain constant value $\varepsilon_d(\tau)_{\max}$ which is parameter $\varepsilon_d(\infty)$, i.e., it can be stated that at a certain time, the further course of the function $\varepsilon(\tau)$ will depend only on $\varepsilon_\eta(\tau)$, and the contribution of the creep $\varepsilon_d(\tau)$ to the total deformation is restricted by the constant value $\varepsilon_d(\infty)$. A characteristic point in this case will be $\varepsilon(\tau_{\mathrm{lim}})$, where for time τ_{lim} the values of the components $\varepsilon_\eta(\tau)$ and $\varepsilon_d(\tau)$ will be equal. Let us find this point.

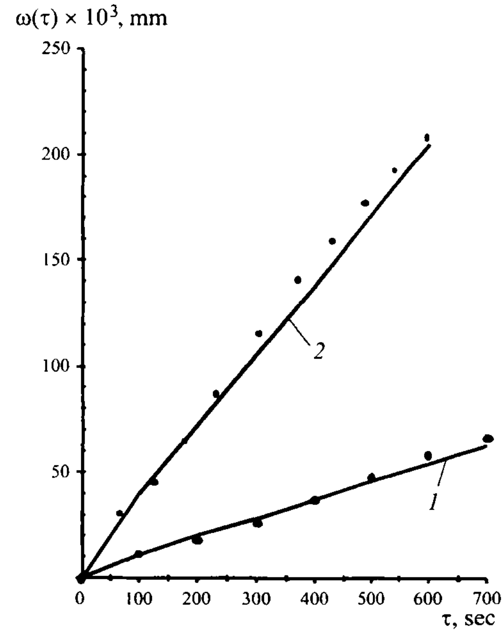

Fig. 3. Variation in beam flexure. Solid curves) estimated values; dots) experimental data; *1*) at temperature of 640°C, σ(0) = 16.29 kPa; *2*) at temperature of 660°C, σ(0) = 17.09 kPa.

Assuming that the value of $\varepsilon_d(\infty)$ for constant temperature and loading conditions is constant, τ_{lim} can be calculated, provided that the function

$$f(t) = 1 - M_\varepsilon(t)$$

at this point has a known value. The value of function $f(\tau)$ can be theoretically determined with sufficiently high precision. Since the curve has substantial curvature and rapidly approaches an asymptote, it can be stated that the point τ_{lim} will be positioned within the interval of the function with values above 0.95 when the further effect of $\varepsilon_d(\tau)$ will be minimum. By denoting the accepted value of $f(\tau)$ as C, it can be written:

$$\tau_{\mathrm{lim}} = \tau_r(-\ln(1 - C))^{1/b}.$$

After finding τ_{lim}, it is possible to determine $\varepsilon_d(\infty)$ since for time τ_{lim}, the values of $\varepsilon_d(\tau)$ and $\varepsilon_\eta(\tau)$ will be equal (calculated in the same coordinate system):

$$\varepsilon_d(\infty) = \frac{\sigma(0)\tau_p}{\eta(1 - M_\varepsilon(\tau_p))}.$$

Using the described method, the current flexure values of beams made of window pane glass were calculated for tem-

peratures of 540 and 660°C (Fig. 3). The sizes of the beams in the calculations corresponded to the values used in the experiments. At 640°C, $l = 20$ mm, $b = 4.71$ mm, $h = 3.12$ mm; at 660°C, $l = 24$ mm, $b = 4.678$ mm, $h = 3.415$ mm. In addition to that, the value of the initial stress $\sigma(0)$ of the beam was taken into account.

The main criterion for verification of the model adequacy is the root-mean-square deviation calculated from the formula:

$$\sigma_0^2 = \sum_1^n \delta_i / (N - k),$$

where δ_i is the deviation of the estimated values from the experimental values; N is the number of measurements; k is the number of the degrees of freedom in the model. For the case considered, $k = 3$.

As a result of the performed calculations, the following root-mean-square deviations were obtained: at the temperature of 660°C, $\sigma = 17.09$ kPa, $\sigma_0 = 2{,}55$; at the temperature of 640°C, $\sigma = 16.29$ kPa, $\sigma_0 = 2.176$.

As can be seen from the results obtained, the root-mean-square deviations are rather low, which is evidence of the adequacy of the proposed model to the actual deformation process in the glass beam.

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